

Introduction to Statistical Inference, 2026

Full Marks: 30 Time : 2.5 hrs

Question No. 1 is mandatory. Do as many as you want from the rest. The maximum you can score is 30.

- The following data shows the effect of a new medicine on patients with primary and secondary hypertension (measured in the unit mm Hg). Carry out suitable test to check if the medicine works better on patients with primary hypertension than the secondary. Consider the maximum allowable type – I error to be fixed at 5%.

Primary Hypertension (mm Hg)	Secondary Hypertension (mm Hg)
122	132
134	130
120	120
126	122
124	126
120	124
122	120
130	124
128	126
136	

$$\tau_{0.05} = 1.645, \tau_{0.025} = 1.960, \tau_{0.01} = 2.326, \tau_{0.005} = 2.576,$$

$$t_{0.05, 9} = 1.833, t_{0.025, 9} = 2.262, t_{0.05, 10} = 1.812, t_{0.025, 10} = 2.228, t_{0.05, 17} = 1.740, t_{0.025, 17} = 2.11,$$

$$t_{0.05, 19} = 1.729, t_{0.025, 19} = 2.093 \quad (8)$$

- Suppose a random sample of size n is drawn from a distribution with pdf

$$f_{\theta}(x) = \begin{cases} \frac{1}{\theta_2} e^{-\frac{x-\theta_1}{\theta_2}}, & \theta_1 < x < \infty \\ 0, & \text{otherwise} \end{cases}$$

$$-\infty < \theta_1 < \infty \text{ and } \theta_2 > 0$$

Find the Sufficient Statistics for the parameters. (5)

- Given iid random variables $X_1, X_2, \dots, X_n$ with common pdf

$$f_{\theta}(x) = \begin{cases} \theta e^{-\theta x}, & 0 < x < \infty \\ 0, & \text{otherwise} \end{cases}$$

Find the MLE and unbiased estimator of θ . Check their variances and make your comment. (6)

- Given iid random variables $X_1, X_2, \dots, X_n$ following $\text{bin}(m, p)$, with 'm' known, find the MVUE of $q^m + \binom{m}{1} q^{m-1} p + \binom{m}{2} q^{m-2} p^2$, where, $q = 1-p$. (5)

5. For an iid random sample of size n from Cauchy distribution with parameter θ , check consistency of sample median as an estimator of θ . (6)
6. State and prove the theorem for Cramer Rao Lower Bound. (6)